8. Homework Assignment

Dynamical Systems II

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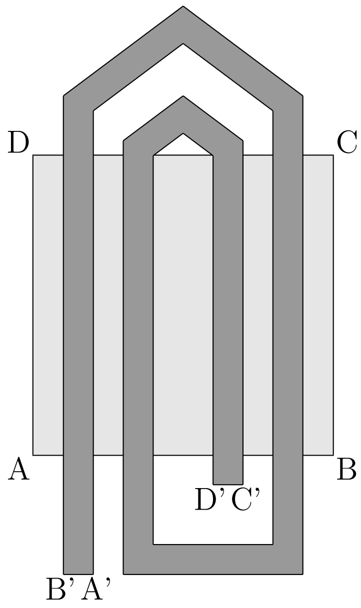
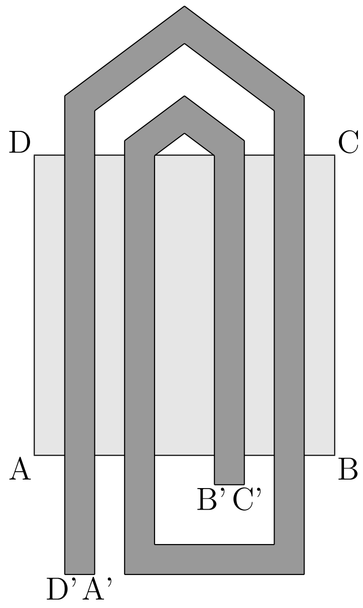
Problem 1: Which of the following "paper-clip" maps gives rise to shift dynamics? (You can assume, that the maps are affine linear, in the regions of intersection.)





Problem 2: Consider the Hénon map

$$\begin{aligned} x_{j+1} &= 1 - \alpha x_j^2 + y_j, \\ y_{j+1} &= \beta x_j. \end{aligned}$$

Find a horseshoe for $1 \ll \alpha$ and $0 < \beta \ll 1$.

For the following two problems consider the situation of a C^0-horseshoe. This means, that the iteration Φ on $\bigcup_{a\in A} V_a$ in the square Q satisfies the assumptions of the theorem about the C^0-horseshoe. Thus, there exists a homeomorphism τ conjugating the shift $\sigma : S \to S$ to $\Phi : I \to I$, on the maximal ϕ - invariant subset $I := \tau(S) \subset \bigcup_{a\in A} V_a$. Let the horizontal and vertical Lipschitz-curves $U(s)$ and $V(s)$ be defined as in class, that is

$$
\begin{aligned}
U(s) &:= \left\{ q \in Q \;\middle|\; \Phi^{-k}(q) \in V_{s_k} \quad \forall k \geq 1 \right\}, \\
V(s) &:= \left\{ q \in Q \;\middle|\; \Phi^{-k}(q) \in V_{s_k} \quad \forall k \leq 0 \right\},
\end{aligned}
$$

for any sequence $s = (s_k)_{k\in\mathbb{Z}} \in S$.

Problem 3: Define the unstable and stable sets of points $p = \tau(s) \in I$ as follows:

$$
\begin{aligned}
W^{\mathrm{u}}(p) &:= \left\{ q \in Q \;\middle|\; \Phi^k(q) \in \bigcup_{a\in A} V_a \quad \forall k \leq -1, \quad \lim_{k\to-\infty} \operatorname{dist}\left(\Phi^k(p), \Phi^k(q)\right) = 0 \right\} \\
W^{\mathrm{s}}(p) &:= \left\{ q \in Q \;\middle|\; \Phi^k(q) \in \bigcup_{a\in A} V_a \quad \forall k \geq 0, \quad \lim_{k\to\infty} \operatorname{dist}\left(\Phi^k(p), \Phi^k(q)\right) = 0 \right\}.
\end{aligned}
$$

Prove or disprove:

(i) $U(s) \subset W^{\mathrm{u}}(p)$ and $V(s) \subset W^{\mathrm{s}}(p)$;

(ii) $W^{\mathrm{u}}(p) \subset U(s)$ and $W^{\mathrm{s}}(p) \subset V(s)$;

Problem 4: For every $s \in S$ is the curve $V(s)$ a graph over the vertical axis. Thus there is a function $v_s(y)$ such that $V(s) := \{(v_s(y), y),\ y \in [0,1]\}$. Prove or disprove, that the curves $V(s)$ depend continuously on their footpoint $(v_s(0), 0)$. That is to say for all $p = \tau(s) \in I$ and all $\epsilon > 0$ exists a $\delta > 0$ such that for all $\tilde{p} = \tau(\tilde{s}) \in I$ with $|v_s(0) - v_{\tilde{s}}(0)| < \delta$ it holds, that $\max_{0\leq y\leq 1} |(v_s(y) - v_{\tilde{s}}(y)| < \epsilon$.